Wave Breaking on an Opposing Current

**US Army Corps
of Engineers**

by Jane McKee Smith

PURPOSE: The Coastal Engineering Technical Note (CETN) herein provides a method to estimate wave breaking on an opposing current, such as at coastal inlet entrances.

PROBLEM: Waves propagating into a tidal inlet will shoal and break because of changes in water depth, focusing by shoals, and interactions with an opposing (ebb) current. On an ebb current, waves steepen because their height increases and length decreases. The wave steepening can intensify wave breaking, causing a navigation hazard and inducing a wave-driven current and sediment transport. Most wave-breaking criteria are based on a maximum ratio of wave height to water depth. But, in regions where waves break because of steepening on an ebb current, the height-to-depth criterion may significantly underestimate wave breaking and overestimate wave height.

CETN IV-9 (Smith 1997) describes the process of wave-current interaction in one dimension and gives the equations for calculating wave-height transformation on an ebb (opposing) or flood (following) current. Extending the discussion in CETN IV-9, the present CETN provides guidance on estimating wave breaking that may occur as waves transform on an opposing current. Before the method for calculating breaking is described, the governing equation, wave-action conservation, is reviewed (see also CETN IV-9), and wave shoaling on a current is discussed.

CONSERVATION OF WAVE ACTION: Wave height in the presence of a current is governed by the conservation of wave action (Jonsson 1990, and others). The one-dimensional conservation of wave action equation is given by:

$$\frac{\partial}{\partial x}\left(\frac{E\left(C_{gr}+U\right)}{\omega_r}\right)=\frac{D}{\omega_r}$$

(1)

where
- x = horizontal coordinate direction (assumed to be the direction of wave propagation)

- E = wave energy (energy is proportional to wave height squared)

- C_{gr} = group velocity relative to current

- U = current velocity

Report Documentation Page

1. REPORT DATE	2. REPORT TYPE	3. DATES COVERED
MAR 1999	**N/A**	**-**

4. TITLE AND SUBTITLE	5a. CONTRACT NUMBER
Wave Breaking on an Opposing Current (CETN IV-17)	5b. GRANT NUMBER
	5c. PROGRAM ELEMENT NUMBER
6. AUTHOR(S)	5d. PROJECT NUMBER
Smith, Jane McKee	5e. TASK NUMBER
	5f. WORK UNIT NUMBER

7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES)	8. PERFORMING ORGANIZATION REPORT NUMBER
US Army Engineer Research and Development Center, Coastal and Hydraulics Laboratory, Vicksburg, MS	

9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES)	10. SPONSOR/MONITOR'S ACRONYM(S)
	11. SPONSOR/MONITOR'S REPORT NUMBER(S)

12. DISTRIBUTION/AVAILABILITY STATEMENT
Approved for public release, distribution unlimited

13. SUPPLEMENTARY NOTES
The original document contains color images.

14. ABSTRACT
This Coastal Engineering Technical Note (CETN) provides a method to estimate wave breaking on an opposing current, such as at coastal inlet entrances.

15. SUBJECT TERMS

16. SECURITY CLASSIFICATION OF:			17. LIMITATION OF ABSTRACT	18. NUMBER OF PAGES	19a. NAME OF RESPONSIBLE PERSON
a. REPORT	b. ABSTRACT	c. THIS PAGE	**UU**	**9**	
unclassified	**unclassified**	**unclassified**			

T_r = angular frequency relative to the current

D = dissipation because of wave breaking

(Other source terms, such as atmospheric input and bottom friction, are neglected here because propagation distances are relatively short, on the order of a few miles or less.)

Consider the situation where waves are traveling into a tidal inlet (in the +x direction) and are opposed by an ebb-tidal current flowing out through the inlet (in the -x direction). As the waves propagate from the ebb shoal into the inlet channel, the ebb-current speed increases (where the flow is confined in the inlet opening), and the wave-current interaction reduces the relative group velocity and increases the relative angular frequency. Thus, the term (C_{gr} + U)/T_r decreases in size. To balance this decrease, the energy E increases through shoaling, and/or the excess energy is dissipated through wave breaking. The depth within the inlet channel may be as great or greater than the ebb shoal, but the wavelength decreases because of the interaction with the opposing current (see CETN IV-9). If the wave dissipation is neglected or underestimated, the wave energy predicted by Equation 1 grows unrealistically large in the inlet. Thus, a method for estimating wave breaking on an opposing current is required.

The discussion of the one-dimensional wave-action equation illustrates the process of wave shoaling on a current, but for general inlet wave-transformation problems, solution of the two-dimensional wave-action equation with refraction is required (see, e.g., Smith, Militello, and Smith 1998 and Holthuijsen, Ris, and Booij 1998). Details on calculating the relative group celerity and relative angular frequency to solve the one-dimensional problem (Equation 1) are given in CETN-IV-9 (Smith 1997).

BREAKING CRITERIA: Miche (1951) specified the maximum monochromatic wave height as a function of wavelength and water depth:

$$H_{\max} = 0.142\,L \tanh kd \qquad\qquad (2)$$

where

H = wave height

L = wavelength

k = wave number ($k = 2\pi/L$)

d = water depth

In deep water, Equation 2 reduces to a maximum wave steepness $H_{max}/L = 0.142$, and in shallow water, it reduces to a maximum height-to-depth ratio $H_{max}/d = 0.88$. This criterion is powerful because it includes both the impacts of depth- and steepness-limited breaking. Equation 2 is implemented as the monochromatic breaking criterion in the one-dimensional wave-current interaction program presented in CETN IV-9.

In the field, waves are irregular, and there is a distribution of wave heights and wave periods. Because of the wave-height variability, many definitions of wave height (statistical and spectral) are possible and are used according to the particular application, such as follows:

$\overline{H}$, the mean wave height.
$H_{1/3}$, the significant wave height (average of the highest one-third of the wave heights).
H_{mo}, the zero-moment wave height (based on the energy in the wave spectrum).
$H_{1/10}$, the average of the highest one-tenth of the wave heights.
$H_{1/20}$, the average of the highest one-twentieth of the wave heights.
$H_{1/100}$, the average of the highest one-hundredth of the wave heights.

Equation 2 was developed for monochromatic waves; thus, it may overpredict or underpredict the wave height (depending on the definition) for irregular waves.

The Coastal Inlets Research Program is evaluating and developing methods to estimate wave conditions in coastal inlets. A product of this effort has been evaluation of breaking criteria based on laboratory data collected in an idealized 1:50-scale inlet model (Smith et al. 1998). The idealized inlet was an opening in a straight beach with rock jetties. The offshore bathymetry was parallel to the straight shoreline, except for an ebb shoal, symmetric about the inlet center line. Water was pumped through the bay area to simulate ebb current through the inlet. The scaled wave conditions were H_{mo} = 3 to 13 ft (0.9 to 4.0 m) and peak period T_p = 5 to 12 sec; the scaled inlet/ebb-shoal water depths were d = 9 to 20 ft (2.7 to 6 m); and the scaled current velocities were U = 0 to -7.2 ft/sec (0 to -2.2 m/sec) (negative velocity denotes a current opposing the waves). From this laboratory data set, the following breaking criteria were determined:

$$\overline{H} = 0.07L \tanh kd \tag{3}$$

$$H_{mo} = 0.10L \tanh kd \tag{4}$$

$$H_{1/3} = 0.10L \tanh kd \tag{5}$$

$$H_{1/10} = 0.12L \tanh kd \tag{6}$$

$$H_{1/20} = 0.13L \tanh kd \tag{7}$$

$$H_{1/100} = 0.15L \tanh kd \tag{8}$$

These criteria represent a maximum value of the wave statistic, based on the local wavelength and water depth. These equations were determined from an average of the highest 10 percent of the parameter $H/(L \tanh kd)$ (where H is defined as each of the wave-height parameters given in Equations 3-8) measured for 47 irregular laboratory wave conditions at 12 inlet/ebb-shoal wave gauges. The breaking-wave heights, especially the higher wave heights, are not well represented by the Rayleigh distribution. Additional discussion of wave breaking on a current and methods to

calculate wave-dissipation rates are given by Ris and Holthuijsen (1996) and Smith, Resio, and Vincent (1997).

Current does not appear explicitly in Equations 3-8 for calculating wave breaking on a current. Instead, current enters through changes in the wavelength in the equations. The ebb current steepens the waves, which induces breaking. For flood current, the wave steepness is reduced, and breaking and dissipation are decreased. The equations are applicable to breaking with or without current. Wave height is determined by limiting the maximum transformed wave height (e.g., calculated using Equation 1 with $D = 0$, the one-dimensional model given in CETN IV-9, or a two-dimensional model) to the value given by the appropriate breaking criterion (Equations 3-8).

EXAMPLES AND DISCUSSION: The wave-current interaction PC program presented in CETN IV-9 has been modified to represent irregular waves as well as monochromatic waves. For irregular waves, the input wave condition is a significant wave height (H_{mo} or $H_{1/3}$), and the breaking criterion applied is Equation 4 (or 5). The option remains to model monochromatic waves using the Miche criterion for breaking. An application of the program is shown in Figure 1 and discussed in the following example.

Example 1

Waves approach an inlet entrance on an ebb current. The channel is long and narrow (thus the one-dimensional assumption is valid). The wave height and the wave steepness in the inlet channel are required to evaluate navigation safety.

Find: H_{mo}, $H_{1/100}$, and wave steepness for irregular waves entering a long, narrow inlet channel on an ebb current.

Given: Offshore wave height $H_{mo} = 8$ ft, and peak period is 6 sec in a water depth $d = 40$ ft. Inlet channel depth $d = 8$ ft, and the ebb current speed is $U = 4$ ft/sec.

Figure 1 shows the user interface for the one-dimensional wave-current interaction program. The input conditions given above have been entered, and the output breaking wave height in the throat is given by the program as $H_{mo} = 4.2$ ft. If the waves are breaking ("yes" flag printed in the last output column), Equations 3 and 6-8 can be used to estimate other wave-height statistics using the wavelength given in the program output ($L = 64$ ft). For example, H_{100} is estimated as

$$H_{1/100} = 0.15\,L\tanh kd = 0.15(64)\tanh\left(\frac{2\mathrm{p}}{64}8\right) = 6.3\,\mathrm{ft}$$

Most often, the significant wave height is used in wave transformation studies, but if considering navigation safety or other aspects of design, the higher waves may be of greater interest. The wave steepness is given in Figure 1, based on H_{mo}, as 0.0656. Based on $H_{1/100}$, the steepness is 0.0984 (= 6.3 ft/64.0 ft).

Wave Current Interaction

Done

Input

Save

┌─ Units ──────────────────────────────┐
│ • American customary   ○ Metric   │
└───────────────────────────────────────┘

Help

┌─ Wave Type ──────────────────────────┐
│ ○ Monochromatic   •   Irregular   │
└───────────────────────────────────────┘

┌─ Offshore ──────────────────┐   ┌─ At Inlet ──────────────────┐
│ Wave height (ft):   8.0 │   │   │
│ Wave period (s):   6.0 │   │ Current • Ebb ○ Flood   │
│ Water depth (ft):   40. │   │   │
│ Current   • Ebb ○ Flood   │   │ Ebb Current (ft/s):   4.0 │
│ Speed (ft/s):   0. │   │   │
│   │   │ Water depth (ft):   8.0 │
└─────────────────────────────┘   └─────────────────────────────┘

┌─ Output ──┐
│   │
│ Keep T(s) d(ft) U(ft/s) H(ft) L(ft) H/L C(ft/s) Cr(ft/s) break   │
│   │
│   6.0 40.   N/A   8.00 167.1 .0479 27.85 27.85   │
│   │
│   6.0 8.0 -4.00   4.20 64.0 .0656 10.67 14.67   yes   │
└───┘

Figure 1. One-dimensional wave-current interaction program input and output

Example 2

This example presents results for two wave/current conditions from the idealized inlet laboratory
study. The data have been converted to prototype (field) scale in the figures, using a scale of
1:50. Figure 2 shows a one-dimensional slice of the bathymetry from the deep offshore section
near the wave generator ($x = 0$ ft) to the outer edge of the ebb shoal ($x = 1,000$ ft) and between
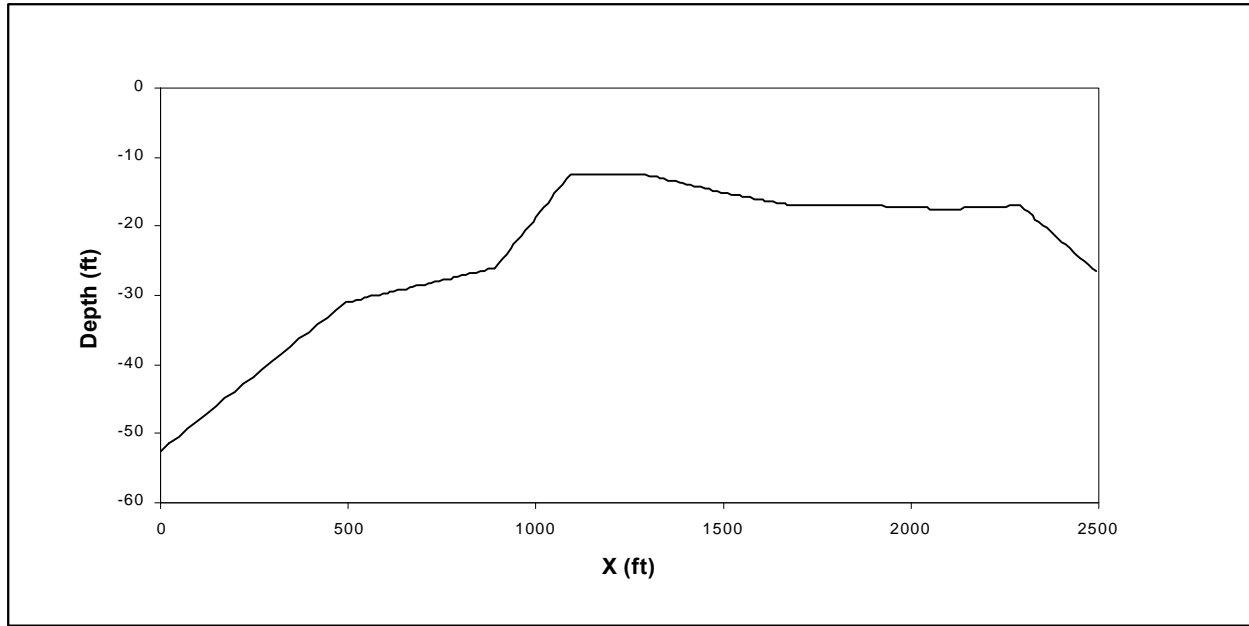the jetties ($x = 1,500$-$2,500$ ft).

Figure 2. Depth profile for Example 2 scaled laboratory case

Find: H_{mo} across the ebb shoal and between the jetties.

Given: Incident wave conditions of $H_{mo} = 9$ ft, and $T_p = 10$ sec and 5 sec; maximum ebb current $U = -6$ ft/sec, and bathymetry given in Figure 2.

The wave heights were modeled by applying one-dimensional wave-current interaction using the incident wave condition (where $U = 0$) and local measurements of water depth and current speed at each computation point. Wave height at each point is limited by the breaking criterion given in Equation 4. The wave-height results are plotted in Figures 3 and 4. In addition, wave height calculated using one-dimensional wave-current interaction, but limited by applying a depth criterion:

$$H_{mo} = 0.6d \qquad (9)$$

is also plotted in the figures for reference. Equation 9 is a typical depth-limited breaking criterion for a spectra wave model (note that the coefficient is less than the typical monochromatic value of 0.78).

These two cases reflect weak and strong wave-current interaction. Weak interaction is illustrated in Figure 3 with incident wave conditions $H_{mo} = 9$ ft and peak period of 10 sec and maximum current of -6 ft/sec (negative indicates ebb flow). Wave-current interaction shoals waves with short wave periods more strongly than waves with long periods, but depth-induced shoaling is greater for long-period waves. The maximum velocity occurs between the jetties ($x > 1,500$ ft) and decreases offshore. The wave-current interaction is weak in this case because the wave
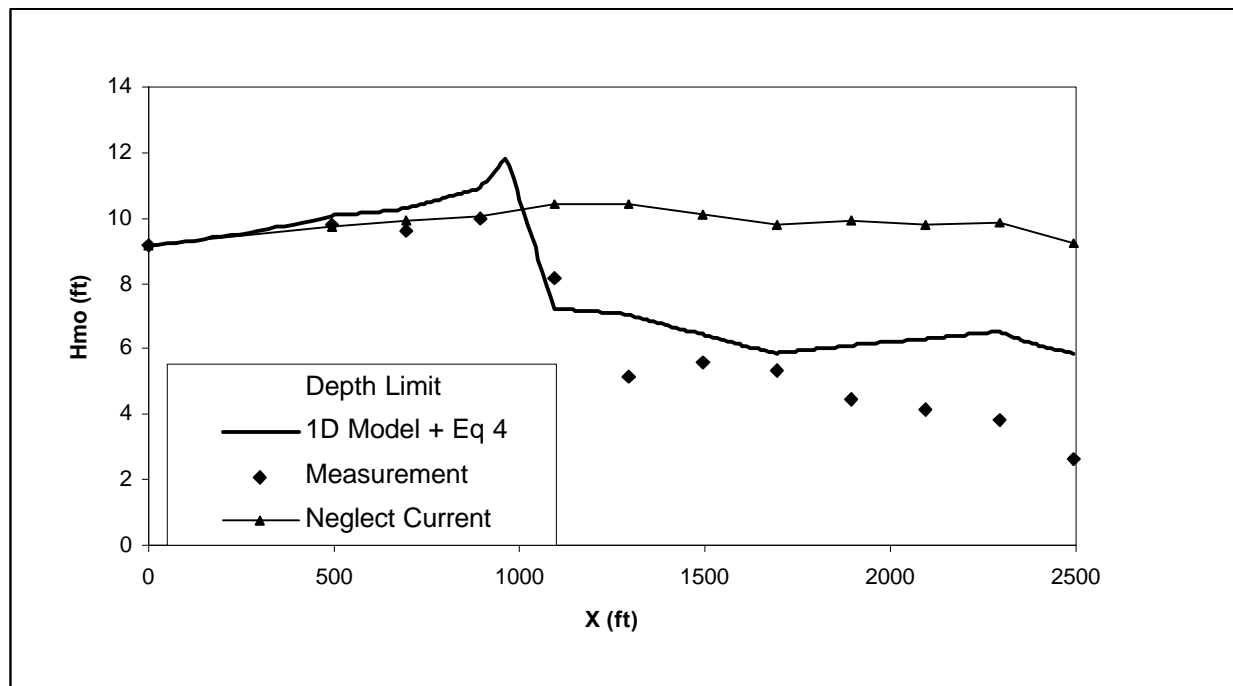
Figure 3. Application of breaking criteria to laboratory measurements (weak wave-current interaction)
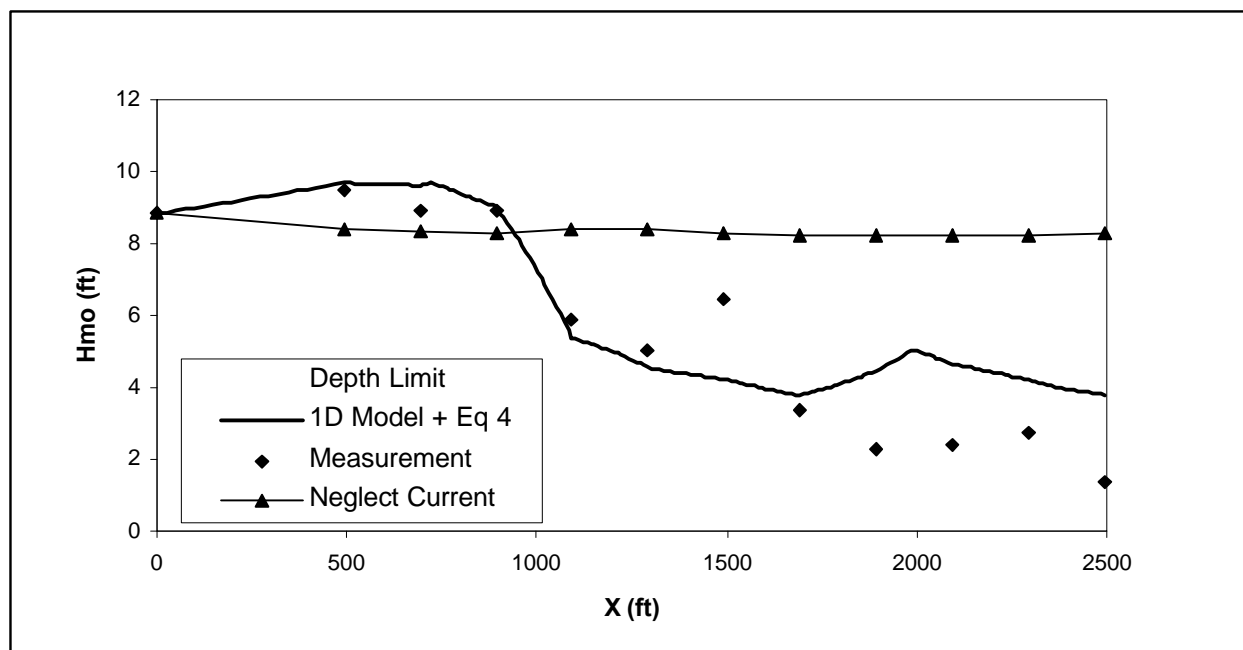

Figure 4. Application of breaking criteria to laboratory measurements (strong wave-current interaction)

period is relatively long, even though the current is strong. The change in wavelength because of the current is relatively small (about 10 percent), and thus shoaling because of wave-current interaction is also small. For the weak interaction cases, the breaking criterion given by Equation 4 acts much like the depth-limited criterion (Equation 9). For reference, an additional curve is included that shows the wave-height transformation for the case where current is neglected. Because the depth variation is relatively small, there is little wave-height variation without wave-current interaction. But, even in the case of weak wave-current interaction, the wave height is significantly overpredicted by neglecting the current and current-induced breaking. In addition to wave height and breaking status, the wave-current interaction program also provides wavelength, wave steepness (H/L), wave celerity (C), and wave celerity relative to the current (C_r). Changes in wave steepness can be used to evaluate navigability at a coastal entrance.

Strong interaction is illustrated in Figure 4 with incident wave conditions H_{mo} = 9 ft and peak period of 5 sec, and maximum current of -6 ft/sec. The wave-current interaction is strong in this case because the wave period is relatively short and current is strong. The waves are close to being blocked by the current. Blocking occurs if the current is so strong that it stops waves from propagating into the inlet (there is no solution to the wave-dispersion equation) and the wave energy is dissipated by breaking or reflected offshore. The shortening of wavelength because of the current is significant (about 50 percent), and thus shoaling because of wave-current interaction is also large. The waves are breaking for $x > 750$ ft because of steepening of the waves (solid curve). For strong interaction cases, limiting wave height using the depth-limited criterion (dashed curve) performs poorly (at $x = 2,300$ ft, the predicted wave height is 10 ft, and the measured wave height is less than 2 ft).

ADDITIONAL INFORMATION:

For additional information, contact Dr. Jane McKee Smith, Coastal Processes Branch, Coastal Sediments and Engineering Division, Coastal and Hydraulics Laboratory, U.S. Army Engineer Research and Development Center, Vicksburg, MS (Voice: (601)634-2079, FAX: (601) 634-4314, e-mail: *jm.smith@cerc.wes.army.mil).* This technical note should be cited as follows:

> Smith, J. M. (1999). "Wave breaking on an opposing current," Coastal
> Engineering Technical Note CETN IV-17, U.S. Army Engineer Research and
> Development Center, Vicksburg, MS. *http://bigfoot.wes.army.mil/cetn.index.html*

REFERENCES:

Holthuijsen, L. H., Ris, R. C., and Booij, N. (1998). "A verification of the third-generation wave model 'SWAN.'" *Proc., 5th International Workshop on Wave Hindcasting and Forecasting.* Environment Canada, 223-230.

Jonsson, I. G. (1990). "Wave-current interactions." *The sea.* Chapter 7, B. Le Méhauté and D. Hanes, ed., John Wiley & Sons, New York, 65-120.

Miche, M. (1951). "Le Pouvoir Reflechissant des Ouvrages Maritimes Exposes a l' Action de la Houle," *Annals des Ponts et Chaussess*, 121e Annee, 285-319 (translated by Lincoln and Chevron, University of California, Berkeley, Wave Research Laboratory, Series 3, Issue 363, June 1954).

Ris, R. C., and Holthuijsen, L. H. (1996). "Spectral modeling of current induced wave-blocking." *Proc., 25th Coast. Engrg. Conf.* ASCE, 1247-1254.

Smith, J. M. (1997). "One-dimensional wave-current interaction," Coastal Engineering Technical Note CETN IV-9, U.S. Army Engineer Waterways Experiment Station, Coastal and Hydraulics Laboratory, Vicksburg, MS.

Smith, J. M., Militello, A., and Smith, S. J. (1998). "Modeling waves at Ponce de Leon Inlet, Florida." *Proc., 5th International Workshop on Wave Hindcasting and Forecasting*. Environment Canada, 201-214.

Smith, J. M., Resio, D. T., and Vincent, C. L. (1997). "Current-induced breaking at an idealized inlet." *Proc.*, *Coastal Dynamics 97 Conference*. ASCE, 993-1002.

Smith, J. M., Seabergh, W. C., Harkins, G. S., and Briggs, M. J. (1998). "Wave breaking on a current at an idealized inlet," Technical Report CHL-98-31, U.S. Army Engineer Waterways Experiment Station, Coastal and Hydraulics Laboratory, Vicksburg, MS.